FNAL beam test Update comparison of resolutions with geometric mean and error propagation methods

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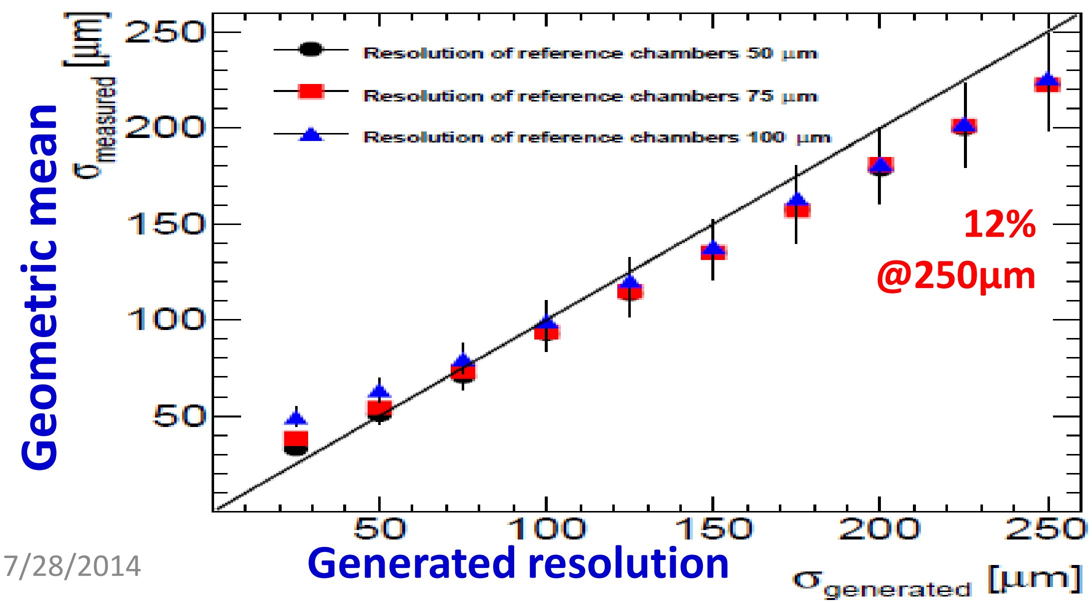
motivation

(1) arXiv:physics/0402054 & arXiv:0705:2210 give theoretical demonstrations on the geometric mean method for resolution calculations. In principle there is no limitation while using this method.

(2) In arXiv:1311.2556, however, it gives some simulation results and claims that the geometric mean method is only applicable when all detectors have similar resolutions.

4. Conclusions

The geometric mean method produces accurate results when the test and reference detectors have the same characteristics. However, when the resolution of the test detector is worse than the reference ones, the result is biased towards better performance. This behaviour is observed in both cases where the test detector is placed inside the reference detectors setup and in the outside area. Finally, it is shown that the distance between the test and reference detectors does not affect the calculated spatial resolution.



- We tried to calculate resolutions with some of the data so that we can compare the results with geometric mean method.
- We did not align the detectors again but just used the existing alignment parameters.

Error propagation method

Step 1. Reconstruct tracks with Tracker 1 & 4, calculate residual of Tracker 2:

$X = \alpha Z + \beta$, where $\alpha = \frac{X_1 - X_4}{Z_1 - Z_4}$ and $\beta = X - \alpha Z$.

Residual of Tracker 2 is

$$R(T_2) = X_{2m} - X_{2p} = X_{2m} - \frac{X_1 - X_4}{Z_1 - Z_4} Z_2 - X_1 + \frac{X_1 - X_4}{Z_1 - Z_4} Z_1$$
$$= X_{2m} + \frac{Z_2 - Z_1}{Z_1 - Z_4} X_4 + \frac{Z_4 - Z_2}{Z_1 - Z_4} X_1$$

Since X_1, X_{2m}, and X_4 are independent variables, and assuming that the standard deviation is the same for all three trackers, and is called only σ, we can simplify the propagation of error equation as follows,

$$\sigma_R^2 = \left(\frac{\partial R}{\partial X_{2m}}\right)^2 \sigma_m^2 + \left(\frac{\partial R}{\partial X_1}\right)^2 \sigma_1^2 + \left(\frac{\partial R}{\partial X_4}\right)^2 \sigma_4^2 =$$
$$\left(\left(\frac{\partial R}{\partial X_{2m}}\right)^2 + \left(\frac{\partial R}{\partial X_1}\right)^2 + \left(\frac{\partial R}{\partial X_4}\right)^2\right)\sigma^2,$$

Note that $\frac{\partial R}{\partial X_{2m}} = 1$, $\frac{\partial R}{\partial X_1} = \frac{Z_4 - Z_2}{Z_1 - Z_4}$ and $\frac{\partial R}{\partial X_4} = \frac{Z_2 - Z_1}{Z_1 - Z_4}$.

By replacing the values of Z_1=0, Z_2=1143.5, and Z_4=3169.5(all in mm) in the above equations, we get σ in terms of σ_R: $\sigma = \frac{\sigma_R}{\sqrt{1.539}} = 0.806\sigma_R$

Error propagation method

Step 2. Reconstruct tracks with Tracker 1, 2 & 4, calculate residual of Tracker 3 (or the GEM):

$$\chi^2(a,b) = \sum_{i=1}^{N} \frac{(X_i - aZ_i - b)^2}{\sigma_i^2}$$

where σ_i is the error on the i^{th} tracker, which is the error of the tracker found in the calculation of step 1. (N=3 (4) when calculating for Tracker 3 (the GEM)).

The minimization of the quantity χ^2 consists of solving the equations: $\frac{\partial \chi^2}{\partial a} = 0$ and $\frac{\partial \chi^2}{\partial b} = 0.$

The derivatives lead to the following set of equations:

$a = \frac{S_1 S_{zx} - S_z S_x}{D}$ and $b = \frac{S_x S_{zz} - S_z S_{zx}}{D}$, where

$D = S_1 S_{zz} - S_z^2, S_1 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}, S_Z = \sum_{i=1}^{N} \frac{Z_i}{\sigma_i^2}, S_X = \sum_{i=1}^{N} \frac{X_i}{\sigma_i^2}, S_{ZX} = \sum_{i=1}^{N} \frac{Z_i X_i}{\sigma_i^2}, S_{ZZ} = \sum_{i=1}^{N} \frac{Z_i^2}{\sigma_i^2}.$

By using the propagation of error, the variance σ_f^2 in the value of any function $f(X_i)$ is

$$\sigma_f^2 = \sum_{i=1}^{N} \sigma_i^2 \left(\frac{\partial f}{\partial X_i}\right)^2$$

$\frac{\partial a}{\partial X_i} = \frac{S_1 Z_i - S_Z}{\sigma_i^2 D}$ and $\frac{\partial b}{\partial X_i} = \frac{S_{ZZ} - S_Z Z_i}{\sigma_i^2 D}$,

Summing over the points, we get $\boldsymbol{\sigma_a^2 = \frac{S_1}{D}, \sigma_b^2 = \frac{S_{ZZ}}{D}}$ **and** $\boldsymbol{Cov(a,b) = \frac{-S_Z}{D}}$.

Error propagation method

Step 2. Reconstruct tracks with Tracker 1, 2 & 4, calculate residual of Tracker 3 (or the GEM):

The residual can be get from:

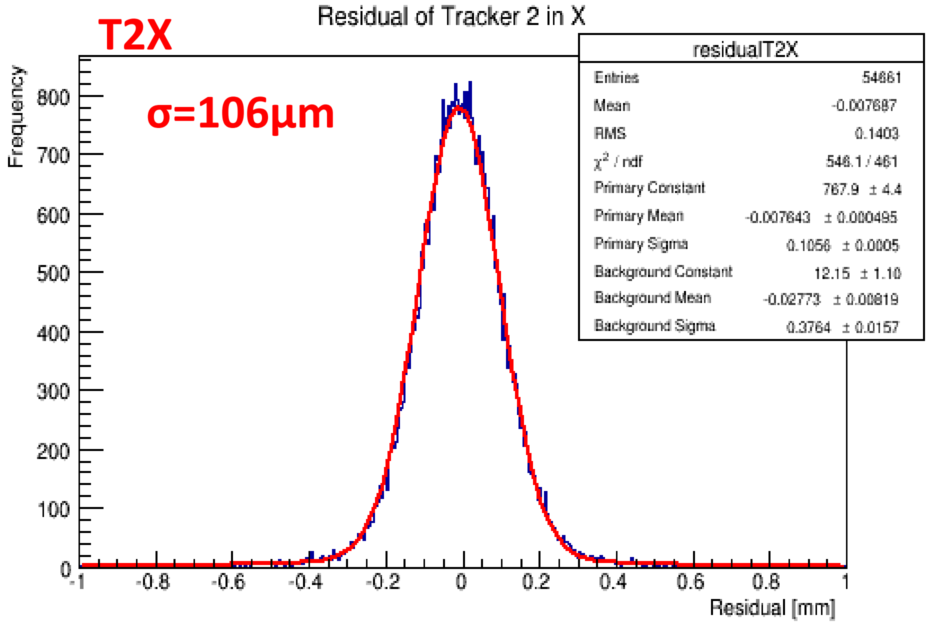
$$R(GEM) = X_m - X_p = X_m - aZ_{GEM} - b$$

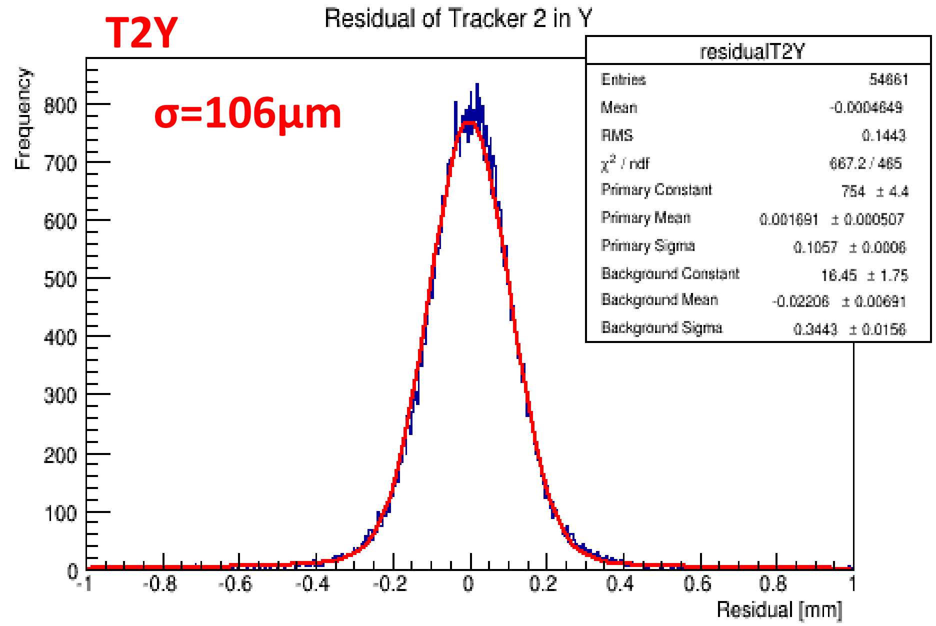
By error propagation,

$$\sigma_R^2 = \left(\frac{\partial R}{\partial X_m}\right)^2 \sigma_{GEM}^2 + \left(\frac{\partial R}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial R}{\partial b}\right)^2 \sigma_b^2 + 2\frac{\partial R}{\partial a}\frac{\partial R}{\partial b} Cov(a,b)$$

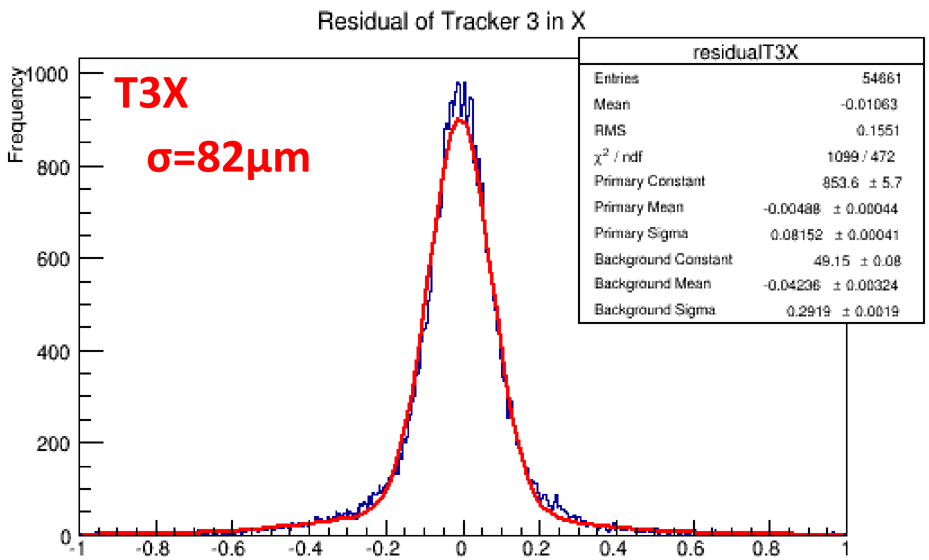
$$\boxed{\sigma_R^2 = \sigma_{GEM}^2 + Z_{GEM}^2\sigma_a^2 + \sigma_b^2 + 2Z_{GEM}Cov(a,b)}$$

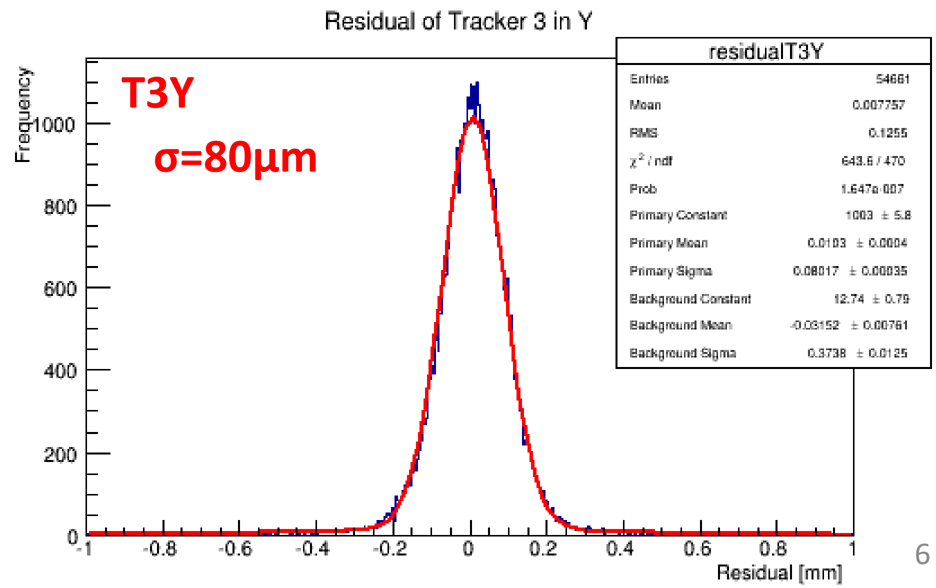
Results for trackers – in Cartesian coordinates



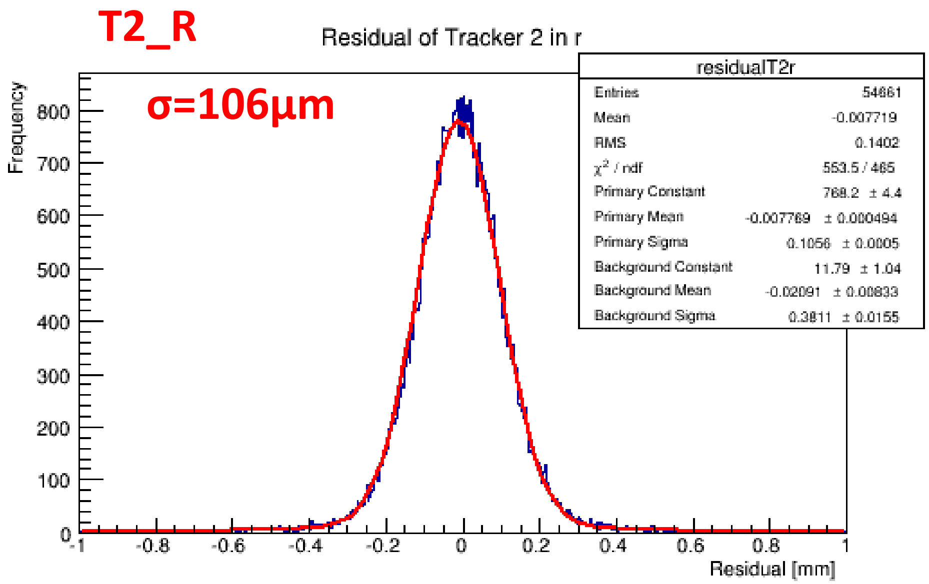


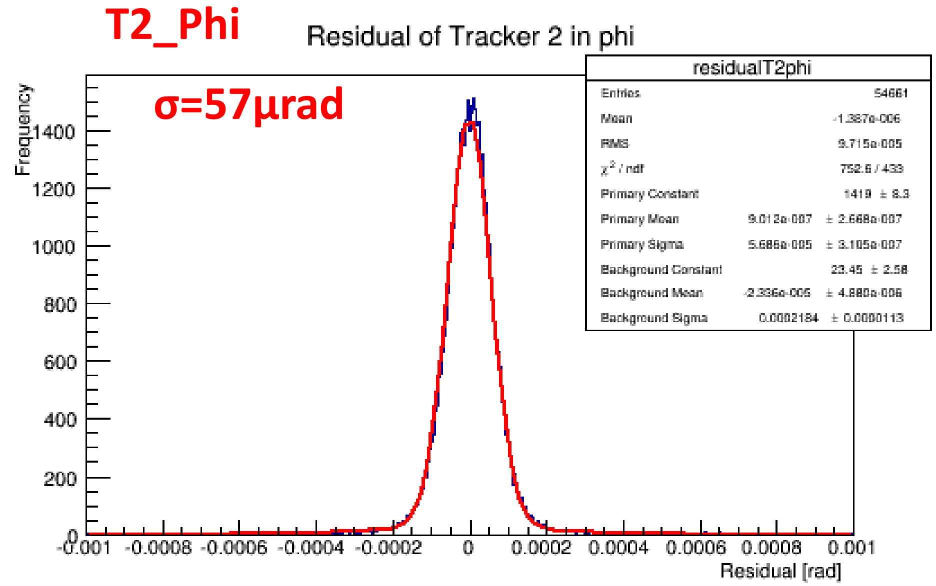
Using Tracker 1 & 4, residual width of Tracker 2 is 106µm, then its resolution is: ~80µm(↑),
and using Tracker 1, 2 & 4, residual width of Tracker 3 is ~80µm, then its resolution is ~45µm(↓).



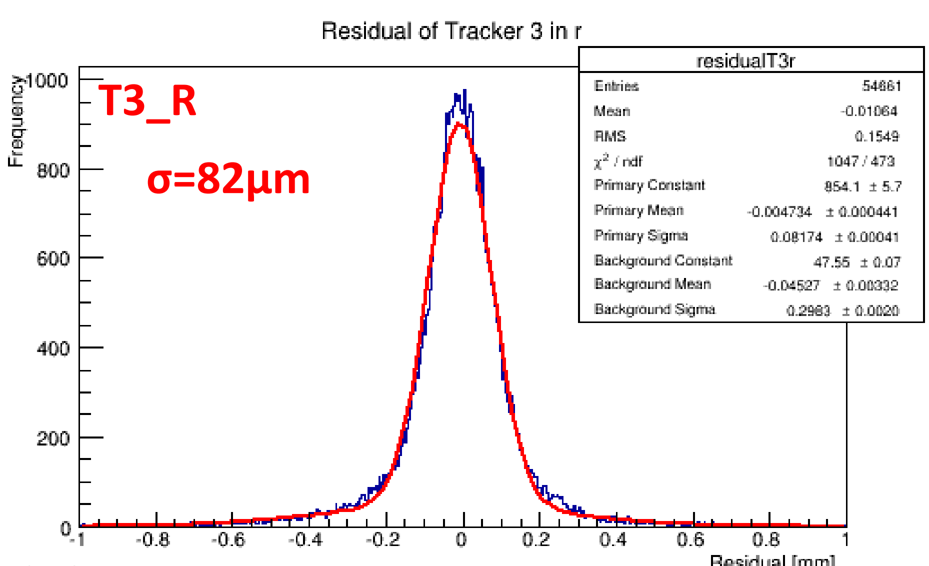


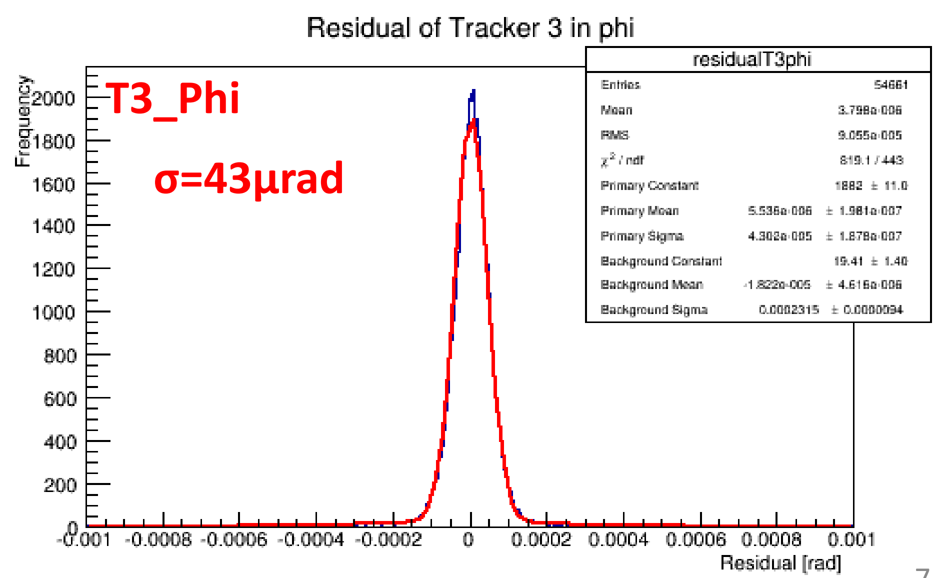
Results for trackers – in polar coordinates



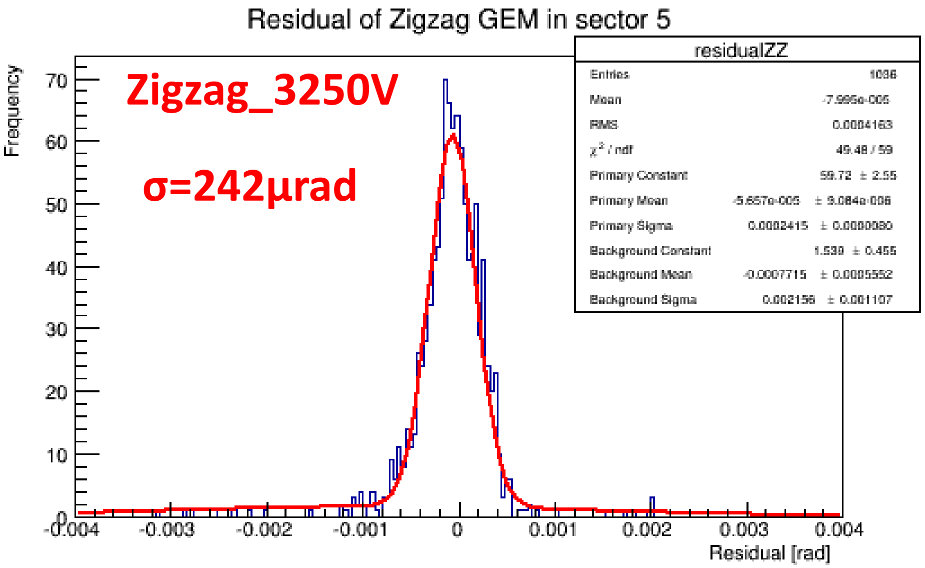


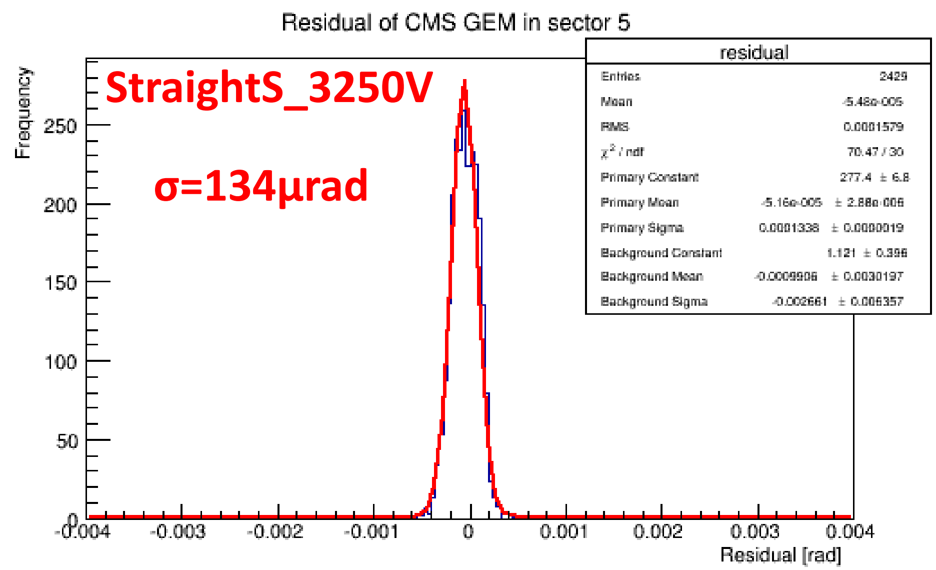
resolutions for Tracker 2 are: ~82μm and 46μrad and for Tracker 3 are ~49μm and 26μrad.



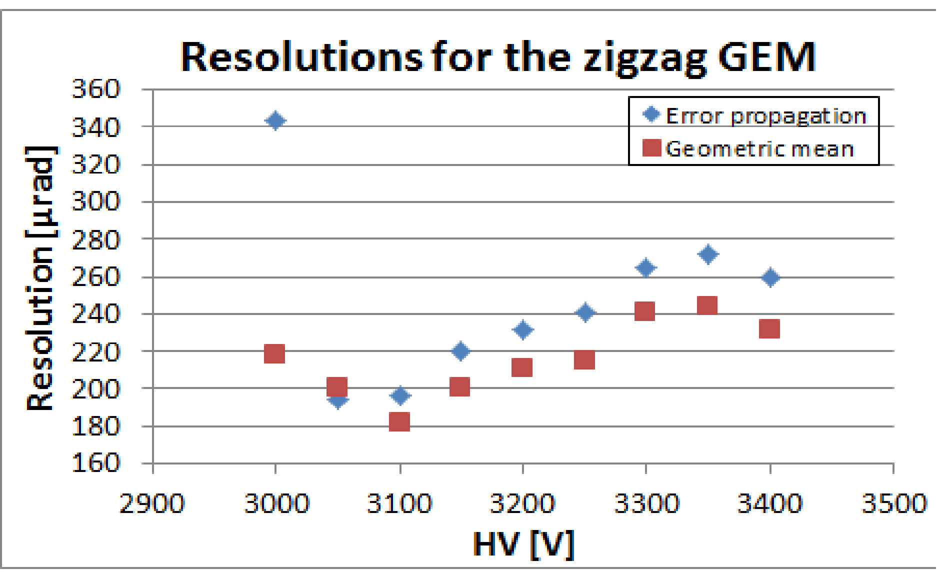


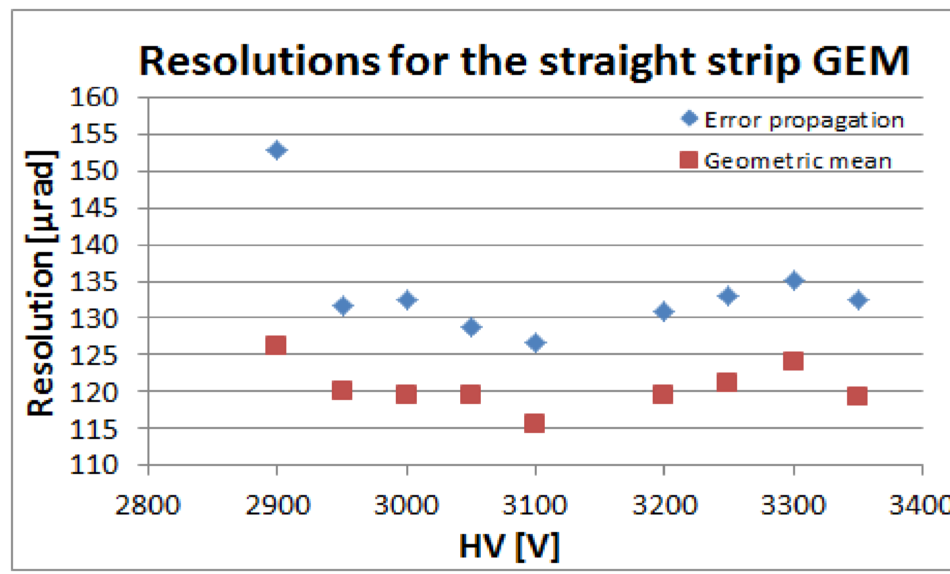
Results for the zigzag GEM and the CMS GEM





Using error propagation, resolution for zigzag is 241μrad and for GE1/1 GEM is 133μrad.





Average difference is 11%

Average difference is 9.7%

Conclusion

Resolutions with geometric mean method are ~10% better than that with error propagation, this is consistent with the simulation results.